Overview of the fields in QFT - Part I (FK8017 HT15)

Real scalar field

Field: $\phi(x) \in \mathbb{R}$

Symmetry: O(1) (no continuous symmetries)

Complex scalar field

Field dofs: $\phi(x), \phi^\dagger(x) \in \mathbb{C}$

Symmetry: $\mathrm{U}(1) \simeq \mathrm{SO}(2)$ = mass degeneracy of the real doublet

Complex field from a real doublet ϕ_1, ϕ_2:

$$\phi(x) = \tfrac{1}{\sqrt{2}}(\phi_1(x) + \mathrm{i}\phi_2(x))$$
$$\phi^\dagger(x) = \tfrac{1}{\sqrt{2}}(\phi_1(x) - \mathrm{i}\phi_2(x))$$

Spinor field

Two-component (Weyl) spinor field

Field dofs: $\chi_a(x) \in \mathbb{C}^2 + \mathrm{hc}$

Symmetry: O(1)

Bispinor field

Four-component spinor field

Field dofs: $\chi_a(x), \psi_a(x) \in \mathbb{C}^2 + \mathrm{hc}$

Symmetry: $\mathrm{U}(1) \simeq \mathrm{SO}(2)$ = mass degeneracy of the 2-spinor doublet

Bispinor from a 2-spinor doublet χ_{1a}, χ_{2a}:

$$\chi_a(x) = \tfrac{1}{\sqrt{2}}(\chi_{1a}(x) + \mathrm{i}\chi_{2a}(x))$$
$$\psi_a(x) = \tfrac{1}{\sqrt{2}}(\chi_{1a}(x) - \mathrm{i}\chi_{2a}(x))$$

Real vector field

Field: $A_\mu(x) \in \mathbb{R}^4$

Gauge symmetry (if free)

Complex vector field

Field dofs: $A_\mu(x), A^\dagger_\mu(x) \in \mathbb{C}^4$

Gauge symmetry (if free)

Free theory = kinetic + opt. mass term

Symmetrized (hermitian) kinetic term:

Real scalar field	Complex scalar field	Spinor field	Bispinor field	Real vector field	Complex vector field
kinetic term ($\mathcal{L} = \mathcal{L}^\dagger$) $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi$	kinetic term ($\mathcal{L} = \mathcal{L}^\dagger$) $\mathcal{L} = \partial_\mu\phi^\dagger\,\partial^\mu\phi$	kinetic term (alt) ($\mathcal{L} = \mathcal{L}^\dagger$) $\mathcal{L} = \frac{1}{2}\Big(\chi^a\,\mathrm{i}\partial_{a\dot a}\overline{\chi}^{\dot a} + \overline{\chi}_{\dot a}\,\mathrm{i}\overline{\partial}^{\dot a a}\chi_a\Big)$	kinetic term (alt) ($\mathcal{L} = \mathcal{L}^\dagger$) $\mathcal{L} = \psi^a\,\mathrm{i}\partial_{a\dot a}\overline{\psi}^{\dot a} + \overline{\psi}_{\dot a}\,\mathrm{i}\overline{\partial}^{\dot a a}\psi_a + \chi^a\,\mathrm{i}\partial_{a\dot a}\overline{\chi}^{\dot a} + \overline{\chi}_{\dot a}\,\mathrm{i}\overline{\partial}^{\dot a a}\chi_a$	kinetic term ($\mathcal{L} = \mathcal{L}$) $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ where $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$	kinetic term ($\mathcal{L} = \mathcal{L}^\dagger$) $\mathcal{L} = -\frac{1}{2}F^\dagger_{\mu\nu}F^{\mu\nu}$ where $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$

EoM based kinetic term:

Real scalar field	Complex scalar field	Spinor field	Bispinor field	Real vector field	Complex vector field
kinetic term (alt) $\mathcal{L}' = -\frac{1}{2}\phi\,\Box\phi$ $\mathcal{L}' = \mathcal{L} + \frac{1}{2}\partial_\mu(\phi\,\partial^\mu\phi)$	kinetic term (alt) ($\mathcal{L}' \neq \mathcal{L}'^\dagger$) $\mathcal{L}' = -\phi^\dagger\,\Box\phi$ $\mathcal{L}' = \mathcal{L} + \partial_\mu(\phi^\dagger\,\partial^\mu\phi)$	kinetic term ($\mathcal{L}' \neq \mathcal{L}'^\dagger$) $\mathcal{L}' = \chi^a\,\mathrm{i}\partial_{a\dot a}\overline{\chi}^{\dot a}$	kinetic term ($\mathcal{L}' \neq \mathcal{L}'^\dagger$) $\mathcal{L}' = \psi^a\,\mathrm{i}\partial_{a\dot a}\overline{\psi}^{\dot a} + \overline{\chi}_{\dot a}\,\mathrm{i}\overline{\partial}^{\dot a a}\chi_a$	kinetic term (alt) $\mathcal{L}' = \frac{1}{2}A^\mu\big(g_{\mu\nu}\Box - \partial_\mu\partial_\nu\big)A^\nu$ $\mathcal{L}' = \mathcal{L} + \frac{1}{2}\partial_\mu(A_\nu F^{\mu\nu})$	kinetic term (alt) ($\mathcal{L}' \neq \mathcal{L}'^\dagger$) $\mathcal{L}' = A^{\dagger\mu}\big(g_{\mu\nu}\Box - \partial_\mu\partial_\nu\big)A^\nu$ $\mathcal{L}' = \mathcal{L} + \partial_\mu(A^\dagger_\nu F^{\mu\nu})$

Optional quadratic terms (mass and gauge fixing):

Real scalar field	Complex scalar field	Spinor field	Bispinor field	Real vector field	Complex vector field
mass term $-\frac{1}{2}m^2\,\phi^2$	mass term $-\,m^2\,\phi^\dagger\phi$	mass term $-\frac{1}{2}\big(m\chi^a\chi_a + m^*\overline{\chi}_{\dot a}\overline{\chi}^{\dot a}\big)$ $-\frac{1}{2}m(\chi^a\chi_a + \overline{\chi}_{\dot a}\overline{\chi}^{\dot a})$ for $m = m^*$	mass term $-\big(m\psi^a\chi_a + m^*\overline{\chi}_{\dot a}\overline{\psi}^{\dot a}\big)$ $-m(\psi^a\chi_a + \overline{\chi}_{\dot a}\overline{\psi}^{\dot a})$ for $m = m^*$	mass term $+\frac{1}{2}m^2\,A_\mu A^\mu$	mass term $+\,m^2\,A^\dagger_\mu A^\mu$
				gauge fixing term $-\frac{1}{2}\zeta\,(\partial_\mu A^\mu)^2$	gauge fixing term $-\,\zeta\,(\partial_\mu A^{\dagger\mu})(\partial_\nu A^\nu)$

Possible self-interaction terms:

Real scalar field	Complex scalar field	Real vector field	Complex vector field
cubic term $-\frac{1}{3!}\mu\,\phi^3$	cubic term $-\mu\,\phi^\dagger(\phi^\dagger + \phi)\phi$	mixed term $-(A_\mu A^\mu)(\partial_\nu A^\nu)$	mixed terms (+hc) $-(A^\dagger_\mu A^\mu)(\partial_\nu A^\nu)$
quartic term $-\frac{1}{4!}\lambda\,\phi^4$	quartic term $-\,\lambda\,(\phi^\dagger\phi)^2$	quartic term $-\,(A_\mu A^\mu)^2$	quartic term $-\,(A^\dagger_\mu A^\mu)^2$

Note: The self-interaction terms for vector fields lead to inconsistencies unless their coupling constants are precisely chosen on the basis of a special type of symmetry, which must involve several vector fields. This symmetry underlies the non-Ahelian gauge theories.

$\psi_a := \chi_a$

$$\Psi_{\mathrm{M}}(x) = \begin{pmatrix}\chi_a(x)\\ \overline{\chi}^{\dot a}(x)\end{pmatrix}, \quad \Psi^\dagger_{\mathrm{M}} = \begin{pmatrix}\overline{\chi}_{\dot a}\\ \chi_a\end{pmatrix}^{\mathsf{T}},$$
$$\overline{\Psi}_{\mathrm{M}} = \Psi^\dagger_{\mathrm{M}}\beta = \begin{pmatrix}\chi_a\\ \overline{\chi}_{\dot a}\end{pmatrix}^{\mathsf{T}}$$

$$\Psi(x) = \begin{pmatrix}\chi_a(x)\\ \overline{\psi}^{\dot a}(x)\end{pmatrix}, \quad \Psi^\dagger = \begin{pmatrix}\overline{\chi}_{\dot a}\\ \psi^a\end{pmatrix}^{\mathsf{T}},$$
$$\overline{\Psi} = \Psi^\dagger\beta = \begin{pmatrix}\psi^a\\ \overline{\chi}_{\dot a}\end{pmatrix}^{\mathsf{T}},$$
$$\gamma^\mu = \begin{pmatrix}0 & \sigma^\mu_{a\dot b}\\ \overline{\sigma}^{\mu\dot a b} & 0\end{pmatrix}, \quad \beta = \begin{pmatrix}0 & \delta_a{}^b\\ \delta^{\dot a}{}_{\dot b} & 0\end{pmatrix}$$

Free-field examples (with propagators):

Real scalar field	Complex scalar field	Spinor field	Bispinor field	Real vector field	Complex vector field
Massive neutral spin-0 $\mathcal{L} = \frac{1}{2}\phi\,(-\Box - m^2)\phi$	Massive charged spin-0 / KGF $\mathcal{L} = \phi^\dagger(-\Box - m^2)\phi$	Majorana Lagrangian ($\mathcal{L} \neq \mathcal{L}^\dagger$) $\mathcal{L} = \overline{\Psi}_{\mathrm{M}}(\mathrm{i}\gamma^\mu\partial_\mu - m)\Psi_{\mathrm{M}}$	Dirac Lagrangian ($\mathcal{L} \neq \mathcal{L}^\dagger$) $\mathcal{L} = \overline{\Psi}(\mathrm{i}\gamma^\mu\partial_\mu - m)\Psi$ $\mathcal{L}^\dagger = \mathrm{i}\partial_\mu\overline{\Psi}(\gamma^\mu - m)\Psi$ $\mathcal{L}' = \mathrm{Re}\,\mathcal{L} = \frac{1}{2}\overline{\Psi}\,\mathrm{i}\gamma^\mu\overleftrightarrow{\partial}_\mu\Psi - m\overline{\Psi}\Psi$ $= \mathcal{L} - \frac{1}{2}\mathrm{i}\partial_\mu(\overline{\Psi}\gamma^\mu\Psi) = \mathcal{L}'^\dagger$	Massless neutral spin-1 (Maxwell) $\mathcal{L} = \frac{1}{2}A^\mu\Big(g_{\mu\nu}\Box - (1-\zeta)\partial_\mu\partial_\nu\Big)A^\nu$	Massive neutral spin-1 (Proca) $\mathcal{L} = \frac{1}{2}A^\mu\Big(g_{\mu\nu}(\Box + m^2) - \partial_\mu\partial_\nu\Big)A^\nu$
Meson propagator: $\mathrm{i}\Delta_{\mathrm{F}}(k) = \dfrac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\varepsilon^+}$	Meson propagator: $\mathrm{i}\Delta_{\mathrm{F}}(k) = \dfrac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\varepsilon^+}$		Dirac fermion propagator: $\mathrm{i}S_{\mathrm{F}}(p) = \dfrac{\mathrm{i}(\gamma^\mu p_\mu + m)}{p^2 - m^2 + \mathrm{i}\varepsilon^+}$	Photon propagator: $\mathrm{i}D^{\mu\nu}_{\mathrm{F}}(k) = \dfrac{-\mathrm{i}\big(g^{\mu\nu} - \frac{\zeta-1}{\zeta}k^\mu k^\nu\big)}{k^2 + \mathrm{i}\varepsilon^+}$	Massive vector propagator: $\mathrm{i}D^{\mu\nu}_{\mathrm{F}}(k) = \dfrac{-\mathrm{i}\big(g^{\mu\nu} - \frac{1}{m^2}k^\mu k^\nu\big)}{k^2 - m^2 + \mathrm{i}\varepsilon^+}$

$\zeta = 1$: Feynman gauge ($\xi = \zeta^{-1} = 1$)

$\zeta \to \infty$: Landau gauge ($\xi = \zeta^{-1} = 0$)

Stückelberg Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(\partial_\mu\phi + mA_\mu)(\partial^\mu\phi + mA^\mu)$$

Gauge-fixing $\phi = 0$, yields the Proca action.

Possible interaction terms:

Yukawa coupling: $-g\,\overline{\Psi}\Psi\,\phi$

Scalar QED: $+e\,\phi\partial^\mu\phi^\dagger\,A_\mu$ $\quad +e\,(\phi^\dagger\phi)(A_\mu A^\mu)$

(Spinor) QED: $-e\,\overline{\Psi}\gamma^\mu\Psi\,A_\mu$

Yukawa coupling: $-g\,\overline{\Psi}\Psi\,\phi$

(Spinor) QED: $-e\,\overline{\Psi}\gamma^\mu\Psi\,A_\mu$

Scalar QED: $+e\,\phi\partial^\mu\phi^\dagger\,A_\mu$ $\quad +e\,(\phi^\dagger\phi)(A_\mu A^\mu)$

Legend:

$g_{\mu\nu}$: Minkowski metric, diag$(1,-1,-1,-1)$

$\mu, \nu, \ldots$: (World) tensor indices

$\alpha, \beta, \ldots$: Four-component (Dirac) spinor indices

$a, b, \ldots, \dot a, \dot b, \ldots$: Two-component (Weyl) spinor indices

$\partial_{a\dot a} := \sigma^\mu_{a\dot a}\partial_\mu, \quad \overline{\partial}^{\dot a a} := \overline{\sigma}^{\mu\,\dot a a}\partial_\mu$

Prescription for building a consistent Lagrangian:

1. The Lagrangian must be real modulo total derivative (required by CPT invariance).
2. All the added terms must be Lorentz-invariant.
3. All the added terms must be of dimension less or equal M^4.

N.b. Any interaction of higher dimension than M^4 leads to a nonrenormalizable theory. Hence, ignoring all the constants, the dimension of any term must be at most M^4 since the dimension of a Lagrangian density is M^4.

$[S] = M^0$, $[\mathcal{L}] = M^4$, $[\partial_\mu] = M^1$, $[\phi] = [A_\mu] = M^1$, $[\Psi] = [\chi_a] = M^{3/2}$, $[F_{\mu\nu}] = M^2$.